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ON SYMMETRIES IN THE GRAPHIC ART OF HORST BARTNIG

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Abstract—Part of the artist's graphic work appears as the result of experiments with operations such as rotation, translation, and color exchange (occasionally), performed on elementary figures of square shape. By means of nine examples, some theoretical framework underlying the figures is discussed in the light of symmetry. Combinatorial as well as graph-theoretical problems are touched.

Common features of science and art, as well as in the practice of apparently such different professions as scientist and artist, are often denied. Moreover, specious arguments are frequently evaluated with reference to results (work of art *vs.* work of science) and become seemingly supported by specific prerequisites of both fields, like strong rationalism on the one hand and dominance of aesthetic aspects on the other. At least art that is classified as constructivistic, however, is subject to technical, sometimes even abstract (mathematical) influences. Consequently, whether or not constructivistic works belong to art is the question, one such that anyone, as an unbiased viewer, might even conclude that he could do the work as well. Here H. Bartnig (who is discussed in this article), at least as far as his work is involved and, perhaps more importantly, in contrast to many other artists, expresses satisfaction about its impact and his fulfilled intentions[1]. Thus giving a viewer the opportunity to reproduce visual processes and to find the rational core of the artistic means is much more important to him than that somebody becomes defeated by ingeniousness, striving after effects (usually concealed), or means of veiling. The questioning of art was the beginning of Bartnig's philosophy, from where finally he came to nonobjectivity in pictorial art. While considering himself obliged to the constructivistic tradition, especially to Malevich, and so-called "concrete painting" as represented by Van Doesburg, he is said[2] to be closely related to the "visual research"[3]. This study is an attempt to elucidate the theoretical framework underlying his graphic art, its systematic manner and order, if any, and patterns of repetition or variation. Only some such repetitions can be described by symmetry operations. As will be seen, most of them can already be understood by permutations.

We know about artists being aware of symmetry (although not always fully), while designing a work in which they make use of it. Hinterreiter is such an example[4], but Bartnig apparently belongs to another group whose representatives produce symmetric compositions somewhat more randomly. On the one hand, these are products of chance, insofar as a single figure is concerned. On the other hand, the occurrence of nontrivial symmetry elements is a necessary concomitant of complete sets of combinations. The artist, however, will usually not arrive at this by deduction. As an experimentalist in permutations, rotations, and translations as well as the exchange of colours, Bartnig passively produces symmetric patterns, thereby preserving the capability to wonder at and enjoy them as they appear.

The only basic element that persistently pervades Bartnig's graphic art is the square. In 1969 he designed a mechanical model, consisting of square fibreboard plates and alternating in black and white. In its normal position the arrangement approaches a pyramid (in Fig. 1 viewed from above) with symmetry C_{4v} [Fig. 1(a)], while synchronous twisting of the plates against each other lets the reflection plane vanish and thus reduces the symmetry to C_4 [Fig. 1(b)]. Symmetry was probably not in the foreground of this construction. It is an object to be touched. Everyone is allowed to experience how a pattern changes quasidynamically under twisting of the pyramid. If it is in its twisted form, e.g. with constant and small angles between

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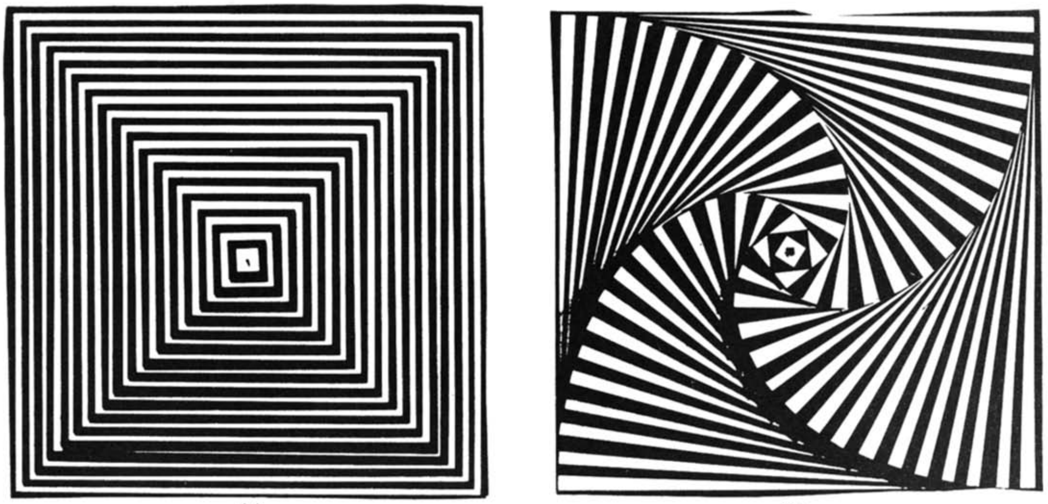
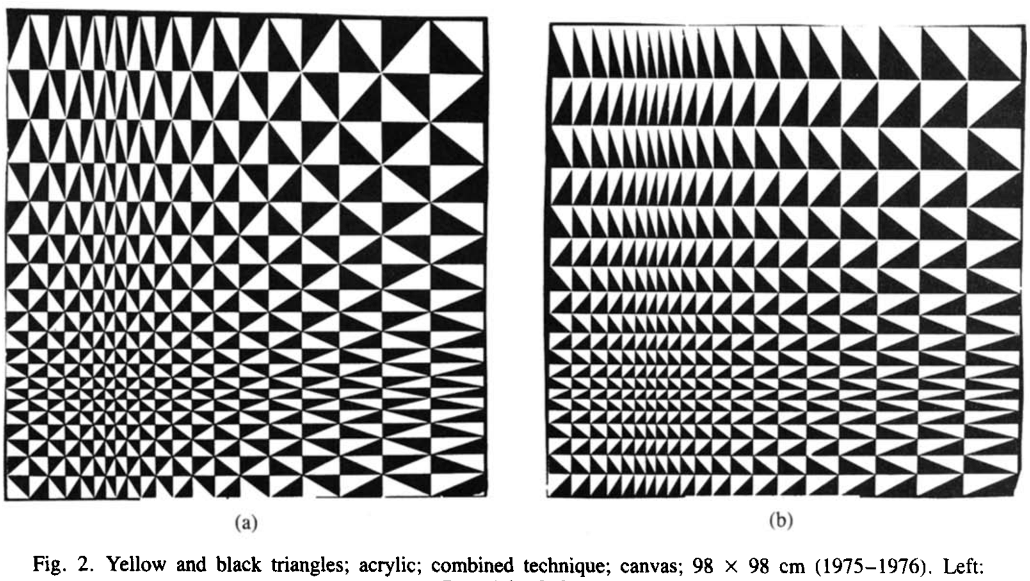


(a) (b)

Fig. 1. Object—changeable squares with central point; fibreboard; 62 × 62 × 20 cm (1969–1974); left: normal position, right: twisted.

neighbouring plates, one seems to see four spirals [Fig. 1(b)]. Such an effect has its explanation in the positions (R_i, ϕ_i) of the corners of each individual square i. They all can be found on Archimedes' spirals (described by the function R_i = const × ϕ_i); therefore one side of the square always approximates the tangent through the point (R_i, ϕ_i) of the curve. Here R_i is the distance from the central point to the corner of the ith square and ϕ_i is the angle of torsion of that square, relative to the normal position [Fig. 1(a)]. The construction built by Bartnig is such that $R_{i+1} - R_i$ = const.

So far we have considered a three-dimensional model, but there are also many examples of drawings where, again, the basic element is a square figure. Together with translations one finds rotations such that either local point symmetry, e.g. C_{2v} or C_4 [Fig. 2(a)] or glide reflection [Fig. 2(b)] occur, if deformations on a nonlinear scale, which the whole system undergoes, are neglected. These examples are selections from the artist's experiments of filling a plane area by drawing repeatedly an elementary figure like , thereby regularly changing its position, primarily with the aim to achieve a specific impression rather than an emphasis on the use of symmetry groups. Thus, for example, one may ask about the origin of curved lines that the viewer can imagine by looking at Fig. 2(a) [but not at Fig. 2(b)] and following the diagonals of the basic squares. As the reduction of the neighbouring elements was chosen by Bartnig such that $a_{i+1}/a_i = a_i/a_{i-1}$, where a_i is the length of a side of the ith square, one "sees" straight



(a) (b)

Fig. 2. Yellow and black triangles; acrylic; combined technique; canvas; 98 × 98 cm (1975–1976). Left: 7/1, right: 7/2.

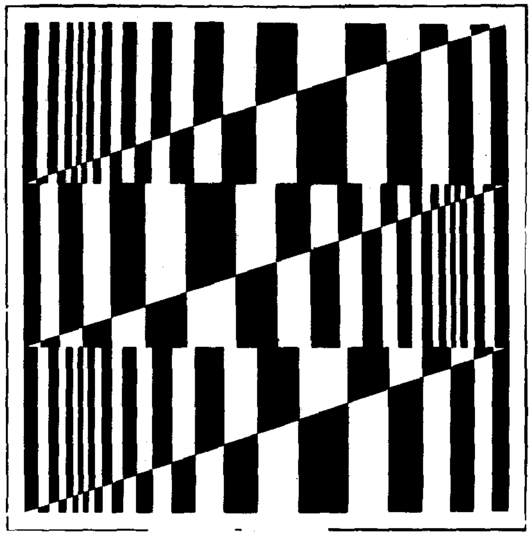


Fig. 3. Composition for Albert Einstein; screen printing; 46 × 46 cm (1979).

lines y = const × x as well as hyperbolae y = const × x^{-1}, where x, y denote a cartesian system, located in the origin of expansion of Fig. 2(a). Figure 3 shows another example of local point symmetry C_2, combined with translation, whereas in Fig. 4 use is made of glide reflection combined with colour exchange.

Figure 5 is the latest example of translation symmetry in Bartnig's art. With screen prints (blue on black ground) of this graphic representation he won the Biennial Prize of the Seventh Norwegian International Print Biennial, held in Fredrikstad in 1984. Although in all cases of Figs. 5(a–f) translation and rotation operations are the same, the general impression depends enormously on the local symmetry and structure of the object that undergoes these operations. For example, compare Figs. 5(a) and 5(d) with their different periodicities, originating from the square with its fourfold axis perpendicular to the paper plane, whereas that of the stroke is only a twofold one. The viewer tends to search for patterns and then to follow these (e.g. the diagonal paths), thereby intuitively interpolating. Thus Figs. 5(d) and 5(e) seem to have similarities at a first glimpse. After precise inspection, however, it becomes obvious that they have none. The reason for the fallacy is that intersecting rectangular figures of twice the size of the square look in this specific arrangement somewhat like the squares themself. With longer rectangles, as in Fig. 5(f), the fallacy vanishes.

In 1975 Bartnig created his "variable systems." In doing so, one basic square figure (the variable) is necessary, occasionally together with its negative (colour exchange). With two such

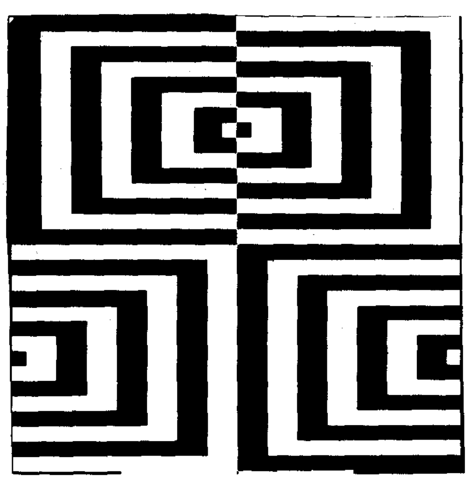
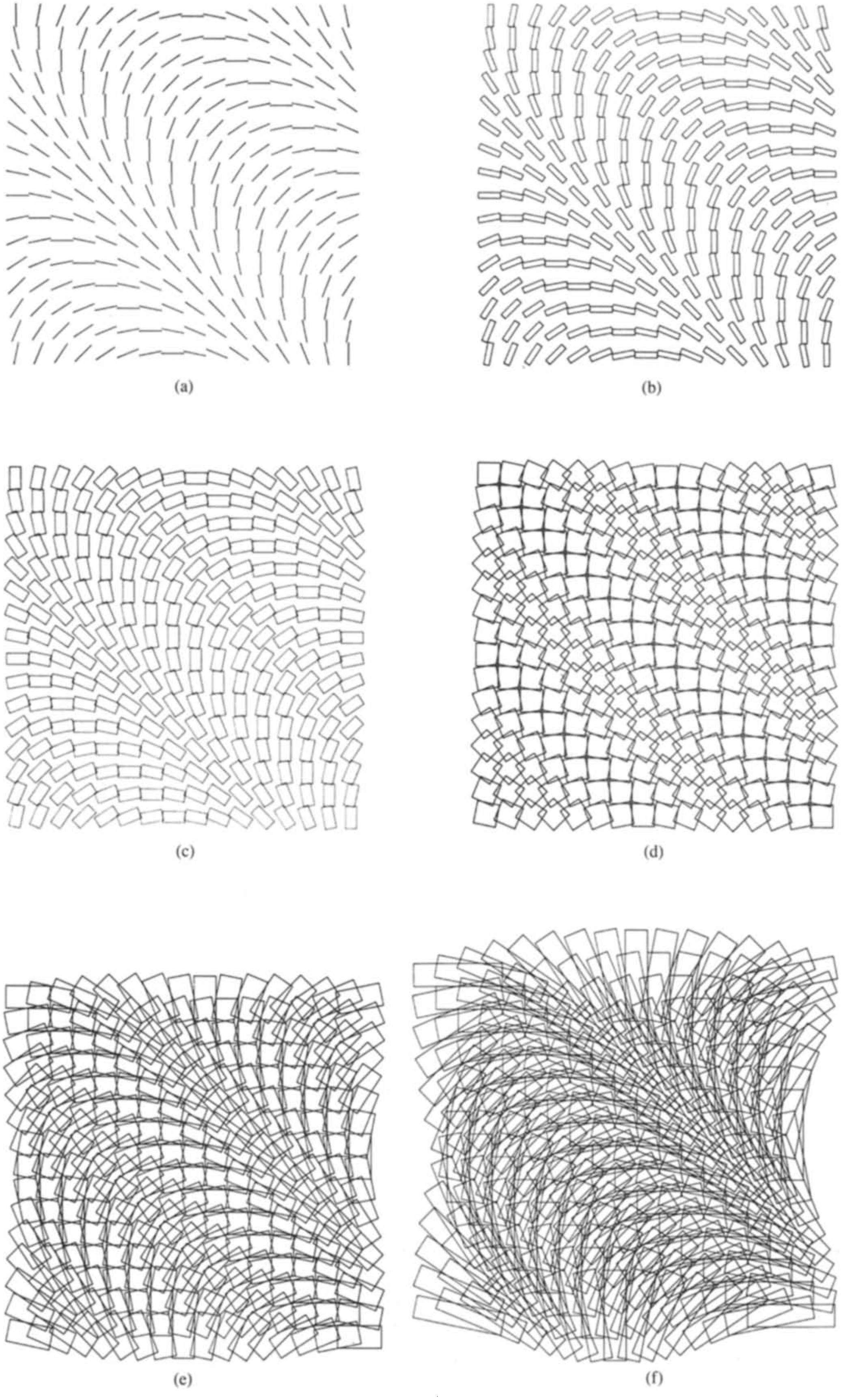


Fig. 4. Black and white squares I (detail); screen printing; 87 × 47 cm (1973–1978).



(a) (b) (c) (d) (e) (f)

Fig. 5. Computer graphics series: 256 strokes (a), 256 small rectangles (b), 256 small rectangles (c), 256 squares (d), 256 rectangles (e), 256 long rectangles (f); between 24 × 24 cm and 28.5 × 28.5 cm (1982–1983) (programmed by D. Garling).

squares rectangles can be composed while four pieces allow supersquares. The artist's intention was to study visual relations between the basic elements in the larger figures, as offered differently in each of them. Therefore he had two intentions, namely the individual composition and the complexity of them all. To achieve this, two variants were offered. One, for example, resulted in the design of a special exhibition catalogue[5], consisting of four separate squares. These allow everyone to practice "constructivistic art," i.e. to attempt his "own" compositions, but one figure after the other. Only the second variant enables us to study all variations simultaneously because here the artist has compiled all possible combinations of four identical squares to form a supersquare. The following analysis of accompanying combinatorial problems touches some elementary symmetry considerations.

It is easily derived that four identical squares can form not more than $4^4 = 256$ different supersquares. The actual limit depends on the symmetry of the basic element, but is further reduced by Bartnig, who excludes all but one supersquare, being interconvertible by rotation and, in some cases, by reflection as well. To put this more precisely, consider for example Fig. 6, which is a first step towards supersquares. Here $4^2 = 16$ rectangles, formed by combining two identical squares in all four positions, were reduced to 10 rotationally invariant figures. This reduction may be derived as follows: Denote the four different rotational positions of a basic square A relative to a fixed rectangle B by

$$A_i \; (i = 0, 1, 2, 3),$$

whereby $\pi i/2$ is the angle of rotation of A. Analogously we have

$$B_i \; (i = 0, 2),$$

with the rectangle B in an external coordinate system. We define

$$B_0 \equiv (A_k, A_l)_0 = A_k, A_l.$$

Then

$$B_2 = (A_k, A_l)_2 = A_{l+2}, A_{k+2},$$

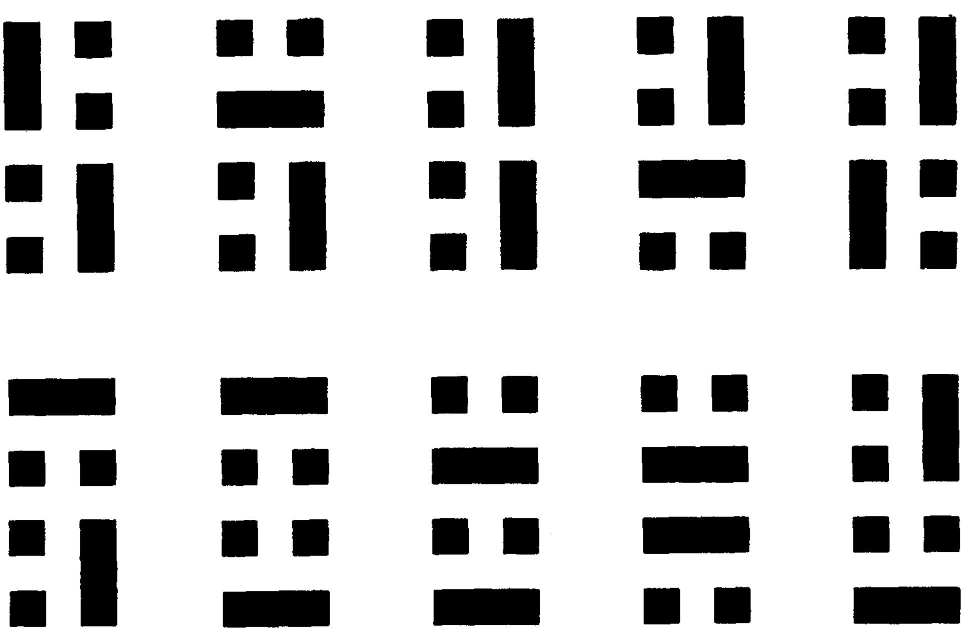


Fig. 6. Relations 3(2); line etching; hand print; 13.5 × 18 cm (1980).

and either B_0 or B_2 has to be eliminated from the set of combinations in order to obtain a set of rotational invariant rectangles (only rotations within the plane of the rectangle allowed). Hence, using the periodicity of rotations, which results in

$$A_i = A_{i \bmod 4}, \quad B_i = B_{i \bmod 4},$$

Table 1 can be set up. The latter four cases therein (separated by a dashed line) satisfy

$$B_2 = (A_k, A_l)_2 = A_k, A_l.$$

This equation formally describes the existence of a C_2 axis and that these figures appear only once in the set of combinations from the very beginning. No other nontrivial element of point symmetry occurs generally, i.e. independently of the presence of symmetry elements in the basic squares.

Let the formation of a supersquare D formally be written as

$$D_0 = \begin{pmatrix} A_j & A_m \\ A_k & A_l \end{pmatrix}_0,$$

and let its four rotational positions

$$D_i(i = 0, 1, 2, 3)$$

correspond to those of the basic squares. Accordingly

$$D_i = D_{i \bmod 4}.$$

Then

$$D_1 = \begin{pmatrix} A_{m+1} & A_{l+1} \\ A_{j+1} & A_{k+1} \end{pmatrix}, \quad D_2 = \begin{pmatrix} A_{l+2} & A_{k+2} \\ A_{m+2} & A_{j+2} \end{pmatrix}, \quad D_3 = \begin{pmatrix} A_{k+3} & A_{j+3} \\ A_{l+3} & A_{m+3} \end{pmatrix}$$

may be proved easily (anticlockwise rotation). Presence of a C_2 axis results in

$$D_0 = D_2, \quad D_1 = D_3,$$

which allows for the (k, j) and (l, m) pairs given in the last four rows of Table 1, provided that $k = l$ and $j = m$. Otherwise

$$k = (j - 1) \bmod 4, \quad l = (k - 1) \bmod 4, \quad m = (l - 1) \bmod 4, \quad j = (m - 1) \bmod 4,$$

while existence of a C_4 axis means

$$D_0 = D_1 = D_2 = D_3.$$

Table 2 lists the 16 symmetrical supersquares. Consequently, the set of rotational invariant 4-membered supersquares is composed of $(4^4 - 16)/4$ figures with a trivial rotational axis C_1, then $(16 - 4)/2$ with C_2 axis, and 4 with C_4 axis; i.e. the total number of supersquares accepted by Bartnig is

$$\frac{4^4 - 16}{4} + \frac{16 - 4}{2} + 4 = 70.$$

As in the case of rectangles (Fig. 6), however, this maximum number may be further reduced if the basic squares have special symmetry elements. For example, if they are identical with their mirror image (reflection plane perpendicular to the plane of the figure) then Bartnig reduces

Table 1. All 16 rectangles (A_k, A_l) composed of two squares. Select one combination in each row to obtain a rotational-invariant set (in the sense of Bartnig). The last four cases have no pendant to be eliminated but are the only ones showing *a priori* a nontrivial element of point symmetry (C_2).

B_0	B_2
A_0, A_0	A_2, A_2
A_1, A_1	A_3, A_3
A_0, A_1	A_3, A_2
A_1, A_0	A_2, A_3
A_1, A_2	A_0, A_3
A_2, A_1	A_3, A_0
A_0, A_2	
A_2, A_0	
A_1, A_3	
A_3, A_1	

the set of supersquares by excluding mirror images, so that the total number finally becomes 39 instead of 70. Otherwise, for example with the absence of a reflection plane in the basic square, this would not work. Two examples are shown in Fig. 7, where the elementary square is  and its negative.

Note also that the whole problem of determining a rotational invariant set of combinations is related to the analysis of chemical reactions. The basic square may represent a molecule with four different binding sites where the supersquare is one of the isomers (actually a tetramer) of the reaction. Despite this analogy, however, reflection planes must be treated differently from the above in a chemically reacting system.

Bartnig also replaced basic squares within the supersquares by their negative, successively one after the other. In this way he arrived at 1044 rotational invariant compositions. Figure 8 shows a selection of 5 from his 70 sheets. The blank spaces therein [Figs. 8(c)–(e)] are a tribute

Table 2. All 16 symmetrical 4-membered supersquares. The four combinations with C_4 axes are separated by a dashed line. Select one combination in each row to obtain a rotational-invariant subset.

D_0	D_1
$\begin{pmatrix} A_0 & A_0 \\ A_2 & A_2 \end{pmatrix}$	$\begin{pmatrix} A_1 & A_3 \\ A_1 & A_3 \end{pmatrix}$
$\begin{pmatrix} A_2 & A_0 \\ A_2 & A_0 \end{pmatrix}$	$\begin{pmatrix} A_1 & A_1 \\ A_3 & A_3 \end{pmatrix}$
$\begin{pmatrix} A_3 & A_0 \\ A_2 & A_1 \end{pmatrix}$	$\begin{pmatrix} A_1 & A_2 \\ A_0 & A_3 \end{pmatrix}$
$\begin{pmatrix} A_0 & A_1 \\ A_3 & A_2 \end{pmatrix}$	$\begin{pmatrix} A_2 & A_3 \\ A_1 & A_0 \end{pmatrix}$
$\begin{pmatrix} A_3 & A_1 \\ A_3 & A_1 \end{pmatrix}$	$\begin{pmatrix} A_2 & A_2 \\ A_0 & A_0 \end{pmatrix}$
$\begin{pmatrix} A_0 & A_2 \\ A_0 & A_2 \end{pmatrix}$	$\begin{pmatrix} A_3 & A_3 \\ A_1 & A_1 \end{pmatrix}$
$\begin{pmatrix} A_1 & A_0 \\ A_2 & A_3 \end{pmatrix}$	
$\begin{pmatrix} A_2 & A_1 \\ A_3 & A_0 \end{pmatrix}$	
$\begin{pmatrix} A_3 & A_2 \\ A_0 & A_1 \end{pmatrix}$	
$\begin{pmatrix} A_0 & A_3 \\ A_1 & A_2 \end{pmatrix}$	

to invariance. In his latest work the artist has changed the arrangement so that now there are 70 supersquares per sheet instead of 16 (see Fig. 9). Although the variations are systematic in a certain sense, they are not governed by symmetry.

Compact presentations of intersecting supersquares of rectangles, such that the set of figures is complete but none appears two or more times, have been tried without success. At least in the case of rectangles it can be proved that any attempt must fail. The task is similar to the graph-theoretical classic "Königsberger Brückenproblem" and therefore closely connected with the existence of so-called Eulerian lines[6]. Making use of a theorem due to Euler it can be proved that, for example, there exists neither a loop nor a line on which the 10 rectangles shown in Fig. 6 can be placed in an overlapping manner. This, however, becomes possible if the degeneracy of the figures is not eliminated; in other words, if all those contained in Table 1 are used. A detailed analysis of these problems would probably be out of place in this study, but will perhaps soon be done elsewhere.

Bartnig's ambitions with his "variable systems" are focused on the generation of supersquare patterns in dependence on the variable design of its elements, with the realization that the visual impression of a supersquare can become either complex or simple as the symmetry of basic squares changes, and also depends on the graphic structure of their edges. So one must consider the match inside a supersquare, taking into account that a viewer usually distinguishes between smooth and abrupt changes and values this aesthetically. On the other hand, it is difficult or even impossible to work out general criteria on how to solve such artistic problems; otherwise they could be treated by nearly anyone who was able to obey the rules, and then it would become questionable whether or not any graphic art would remain. Bartnig, while experimenting with colour exchanges (positive *vs.* negative), easily recognized that this process should not govern the impression of a composition, as perhaps demonstrated in Figs. 9(a)–(f). Contrary to these, Fig. 9(h) is an example of internally well-balanced coloured areas in the basic element (black:white = 1:1). He lays emphasis on the evaluation of new and ingenious patterns, not easily deduced from that of an isolated basic square alone, or its rotation as well as colour exchange. According to his present conviction, the role of highly symmetrical supersquares, as they appear in limited numbers, is the following: Their perfect regularity becomes especially pronounced by an environment of less symmetrical figures, the other supersquares. In a quasidynamical consideration (looking from one to the other) there suddenly rises much clarity and, in a certain sense, despite simplicity, also beauty. The colour exchange, by also increasing the complexity, helps avoiding monotony that otherwise would occur. It may also be considered a perturbation that penetrates the set of supersquares, thereby generating new compositions, among them some new symmetrical ones.

Technically, Bartnig has the image of a computer-graphical artist. This, however, has not been cultivated in this study because it means, in his case, computer-aided generation of some

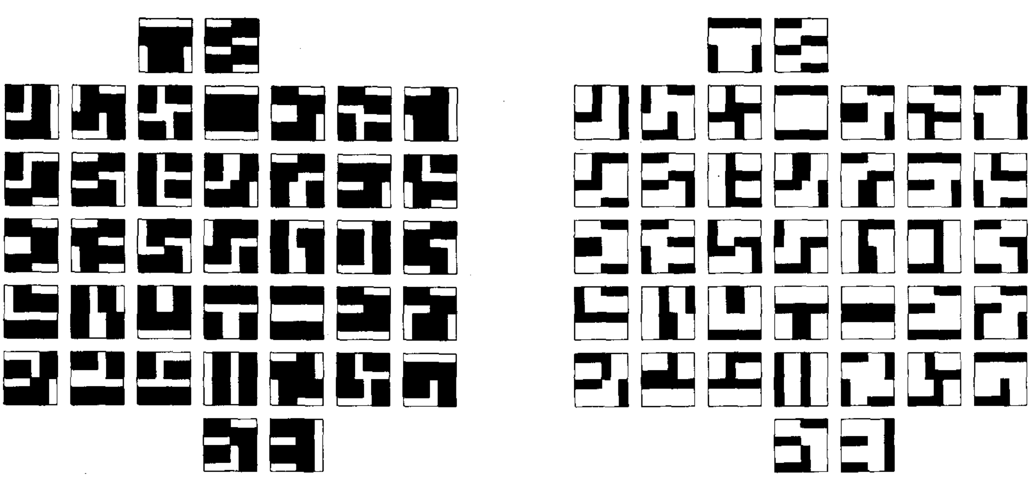


Fig. 7. Seventy eight times four squares with striae; line etching; 12 × 26.5 cm (1982).

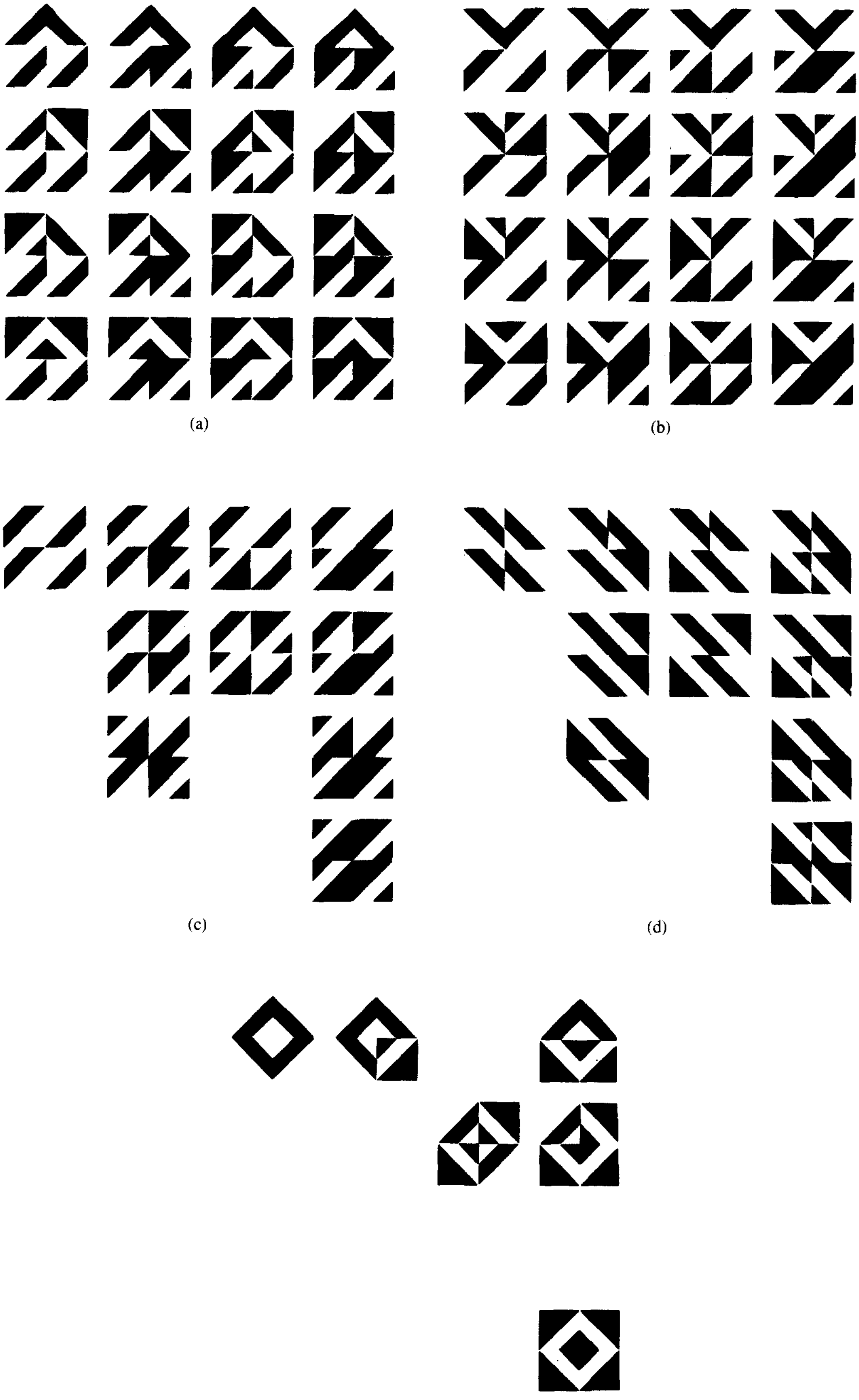


Fig. 8. One thousand fourty four variations; 70 sheets (yellow on white ground); lino-cut; 37.2 × 37.2 cm. Sheet 7, variations 97–112 (a); sheet 34, variations 529–544 (b); sheet 62, variations 971–980 (c); sheet 64, variations 991–1000 (d), sheet 69, variations 1033–1038 (e) (1975–1979).

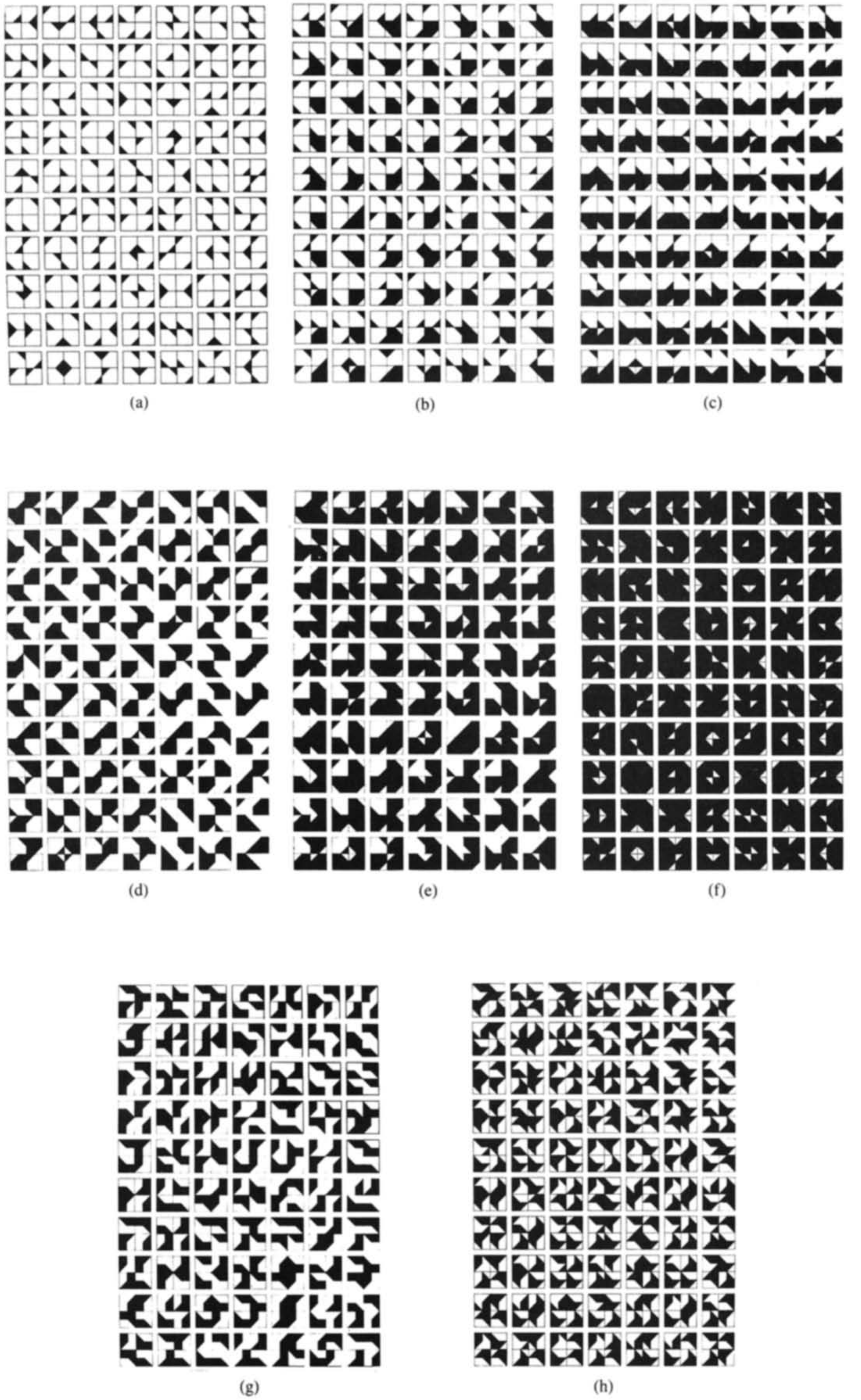


Fig. 9. Three thousand six hundred twenty two variations; 55 sheets; series of computer-supported graphics; line etching; hand print; 36 × 25 cm. Sheet 8 (a), sheet 9 (b), sheet 11 (c), sheet 14 (d), sheet 15 (e), sheet 23 (f), sheet 24 (g), sheet 47 (h) (1984). (Programmed by M. Fischer; adviser: R. Koch.)

primary steps only, e.g. contours (using an X, Y-plotter), or simply a facility for easily achieving completeness of combinations. All these things could otherwise be done by hand but do not necessarily require a new technique. (By the way, the underlying programs were written by others; see, for example, Figs. 5, 9.) With today's term "computer graphics" the reader would have immediately associated colour displays, connected with supercomputers, and perfect real-time "dynamical" compositions, impossible to make with an artist's traditional equipment. Finally, those readers not only interested in the graphic art of Bartnig but also in the whole "constructivistic" scene, in which he has a well-established place, may be referred to [7,8].

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